Prove that $n! > 2^n$ for all integers $n \geq 4$ by mathematical induction. **SCORE: _____ / 12 PTS**

PROOF:

BASIS STEP: $4! = 24$

$2^4 = 16$

$4! > 2^4$

INDUCTIVE STEP: ASSUME $k! > 2^k$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 4$

[PROVE $(k+1)! > 2^{k+1}$]

$(k+1)! = (k+1)k! > (k+1)2^k > 2 \cdot 2^k = 2^{k+1}$

SINCE $k \geq 4$

$k+1 \geq 5$

SO $k+1 > 2$

GRADED BY ME

BY MI, $n! > 2^n$ FOR ALL INTEGERS $n \geq 4$

Find ${}_{300}C_3$. You must show intermediate steps, so do not use the ${}_nC_r$ button on your calculator. **SCORE: _____ / 4 PTS**

$$\frac{300!}{3!\,297!} = \frac{\overset{50}{\cancel{300}} \cdot 299 \cdot 298 \cdot \cancel{297!}}{\cancel{6} \cdot \cancel{297!}} = 4{,}455{,}100$$

(1) (1) (2)

Use the Binomial Theorem to expand and simplify $(3x-4y)^5$. **SCORE: _____ / 7 PTS**

$$1(3x)^5(-4y)^0 + 5(3x)^4(-4y)^1 + 10(3x)^3(-4y)^2 + 10(3x)^2(-4y)^3 + 5(3x)^1(-4y)^4 + 1(3x)^0(-4y)^5$$

$$= 243x^5 - 1620x^4y + 4320x^3y^2 - 5760x^2y^3 + 3840xy^4 - 1024y^5$$

(½) (½) (½) (½) (1) (1) (1) (½) (1) (½)

Write the repeating decimal $0.3\overline{18}$ as a simplified fraction. **NOTE: Only the 18 is repeated.** **SCORE: _____ / 7 PTS**

$$0.3 + 0.018 + 0.0018 + 0.000018$$

$$* \frac{1}{100} \qquad * \frac{1}{100} \qquad (2)$$

$$= \boxed{\frac{3}{10}}^{(1)} + \boxed{\frac{\frac{18}{1000}}{1 - \frac{1}{100}}} (2)$$

$$= \frac{3}{10} + \frac{\frac{18}{1000}}{\frac{99}{100}}$$

$$= \frac{3}{10} + \frac{18}{1000} \cdot \frac{100}{99} = \frac{3}{10} + \boxed{\frac{1}{55}}^{(1)} = \frac{33+2}{110} = \frac{35}{110} = \boxed{\frac{7}{22}}^{(1)}$$

Find the coefficient of $x^4 y^5$ in the expansion of $(2x-5y)^9$. **SCORE: _____ / 5 PTS**

GENERAL TERM = $\binom{9}{r}(2x)^{9-r}(-5y)^r \rightarrow r=5$

$\binom{9}{5}(2x)^4(-5y)^5 = 126\,(16x^4)(-3125y^5)$ ①

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$= -6{,}300{,}000\,x^4y^5$

①